Variation in Mathematics Content Coverage, Instructional Grouping, and Representational Strategies—An Analysis of Three U.S. Kindergarten Mathematics Textbooks

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For several decades, international comparisons of mathematical achievement have indicated that students in the U.S. perform below their peers in many comparable countries (Mullis et al., 2020). Policy makers have also voiced concerns about college and workforce readiness and inconsistent educational expectations across states. In response to this, and other factors, the National Governors Association Center for Best Practices and the Council of Chief State School Officers developed the Common Core State Standards (2010) for English language arts and mathematics in the late 2000s. Early work conducted by the National Council of Teachers of Mathematics (NCTM), including the guidelines for math instruction summarized in Principles and Standards (2000) and Curriculum Focal Points (2006), serve as a basis for many of the standards.

The Common Core State Standards for Mathematics (CCSSM) include standards for both instructional practices (e.g., recommendations for strategic and appropriate use of learning tools) and content (e.g., grade-specific recommendations for the content that should be covered, such as "count to tell the number of objects"). The CCSSM were intended to address a common critique that mathematics instruction in the U.S. covers too many topics at the cost of deep understanding (e.g., Schmidt et al., 2002). They were designed to promote depth over breadth, support students in developing strong conceptual understanding and seeing connections between mathematical practices, improve procedural fluency, and provide opportunities to apply math to solve a broad array of practical problems (Common Core State Standards Initiative, 2013; National Council of Teachers of Mathematics, 2000).

The CCSSM presented an ambitious vision for improving the quality of math instruction in the U.S. However, while the CCSSM outline expectations regarding content and mathematical practices, the standards do not specify exactly how to meet these expectations. For example, the CCSSM specify that children in kindergarten should be able to "Understand that each successive number name refers to a quantity that is one larger," but the standards do not detail how to communicate that understanding. Further, the standards state that "mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace" but do not specify how to develop that proficiency. A key question for policy makers and practitioners is how teachers translate these standards, and standards more generally, into instructional practice. Textbooks likely play a crucial role. Textbooks and other organized resource materials constitute the "potentially implemented" curriculum—the content and activities that a teacher could hypothetically implement (Valverde, 2002). Textbooks help bridge the gap between the *intended* curriculum (the intentions, aims, and goals of a set of standards, for example) and the *implemented* curriculum, or the actual strategies, practices, and activities in which the teacher engages. As such, textbooks often set the stage regarding opportunities to learn mathematics (Son & Diletti, 2017).

Empirical evidence suggests that teachers rely heavily on textbooks to guide instruction. Over 90 percent of elementary school math teachers in the U.S. report using their district's designated curriculum in over half of their lessons, with 76% indicating that they used the designed curriculum in "nearly all" of their lessons (Blazar et al., 2019). Textbooks often determine *what* content is covered, *when* it is introduced, and *how* that content is presented (Van de Walle et al., 2010). High school teachers typically cover around 70% of the material in a textbook (McNaught et al., 2010). Teachers also rely heavily on topic sequencing in textbooks (Flanders, 1994) and content that is not covered in textbooks is generally not taught (e.g., Flanders, 1994; Tarr, 2006). Achievement differences across countries are directly related to what the curricula in those countries cover (Schmidt et al., 2002). As a result, textbooks likely serve as a key mediator in the implementation of standards across classrooms. The extent to which various textbooks interpret standards differently, or emphasize different material or approaches to delivering content, likely has important implications for teaching and learning.

Not only do textbooks potentially mediate the translation of standards like CCSSM into practice, textbooks also potentially influence student achievement. Several studies, utilizing varied methods to compare the effectiveness of early math curricula, have shown that textbooks have an impact on students' mathematics achievement and understanding (Agodini & Harris, 2010; Bhatt & Koedel, 2012; Bhatt et al., 2013; Kane et al., 2016; Koedel et al., 2017). The more effective curricula in each study increased average student achievement by between 0.05 to 0.30 standard deviations above the comparison curricula. While these studies indicate that some textbooks are more effective than others are, they do not help us understand what is driving differential achievement outcomes across textbooks.

Purpose

The central goal of the current study is to compare and contrast three widely used, CCSSM-aligned kindergarten mathematics textbooks along three key dimensions that substantially impact student learning in mathematics, particularly for young children. These include: a) the mathematical content that is covered, (b) the frequency of opportunities for small-group instruction and peer interaction, and (c) the use of concrete and pictorial representation.

We focus on kindergarten mathematics because kindergarten is a formative year for students, marking the transition to formal school, and because evidence indicates that mathematics learning in kindergarten predicts later math outcomes (e.g., Claessens et al., 2009). Despite the potential importance of kindergarten mathematics teaching and learning, research on kindergarten mathematics textbooks is limited, with most curricular analyses focusing on later grades (e.g., 1-5), and particularly grades 3-5 where state testing is ubiquitous (e.g., Agodini & Harris, 2010; Bhatt & Kodell, 2012; Polikoff, 2015).

Specifically, the study explores the following research questions:

- How do the three textbooks vary in their content coverage and sequencing of mathematical topics?
- How do the three textbooks vary in their guidance for instructional grouping?
- How do the three textbooks vary in their specification for concrete and/or pictorial representation?

We review the literature on each of the three dimensions below.

Mathematical Content Coverage

The content to which students are exposed matters—students do not learn material to which they are not exposed and substantial theory and research point to the role that content plays in students' opportunities to learn. Prior research on mathematics content coverage in kindergarten (most of which relies on data collected prior to the implementation of CCSSM) has shown that teachers place considerable emphasis on basic content, such as simple counting and knowledge of basic shapes, which most students have already mastered when they enter kindergarten (Engel et al., 2013; Engel et al., 2016). More importantly, this research finds that exposure to more advanced content, such as addition and subtraction, is associated with larger learning gains for kindergartners (Engel et al., 2013; Engel et al., 2016). To the extent that kindergarten textbooks emphasize more or less advanced content, even among those that are aligned to the CCSSM, they likely mediate the opportunities students have to be exposed to more advanced math material. Students have higher test scores when the textbooks that they are using provide the opportunity to engage in tasks that demand higher levels of understanding (Hadar, 2017).

The *range* of content to which students are exposed also matters for their long-term learning (Clements & Sarama, 2021). While the CCSSM indicate that instructional time should favor numbers and operations over other topics, both geometry and measurement are part of the learning standards for kindergarten (Achieve the Common Core, n.d.). Shape is a fundamental concept in cognitive development, and practices such as composing and decomposing shapes help to build a foundation for understanding part-whole relationships and fractions (Clements & Sarama, 2021). Measurement is an important real-world area of math, and concepts of length, area, and capacity connect geometric concepts to numbers (Clements & Sarama, 2021). Eighth grade outcomes are predicted by early skills in both pattern recognition and measurement (Claessens & Engel, 2013). Despite their importance, U.S. students' performance in geometry and measurement lags substantially behind their peers from other countries (Clements & Sarama, 2021; Mullis et al., 2020; Schmidt et al., 2002). Children are likely to benefit from exposure to a broad range of mathematical practices in addition to a strong emphasis on numbers and operations.

Finally, the sequence in which textbooks introduce and review topics influences students' opportunities to learn. Some textbooks use a spiraling approach to delivering content, introducing and reintroducing mathematical topics with gradually increasing complexity throughout the school year. Others follow a mastery approach and prioritize "mastery" of one topic before moving to another.

Countries that outperform the U.S. on international assessments of mathematics typically employ a mastery approach to mathematics instruction, which prioritizes knowing one aspect of a topic well before moving to the next (Jerrim & Vignoles, 2016; Schmidt et al., 2002). As a result, the CCSSM were designed to emphasize depth over breath—placing a greater emphasis on understanding fewer topics. The potential downside to a mastery approach is that topics covered towards the end of a textbook may not be taught at all if teachers are unable to complete the entire set of textbook units before the end of the school year.

Small Group Instruction and Interaction with Peers

Young children learn best when they are able to work in small groups and engage with their peers (Sarama & Clements, 2009; Skinner, 2018; Stright & Supplee, 2002; van Oers, 2010). Many of the mathematical practices prioritized by the CCSSM (e.g., constructing mathematical arguments and analyzing the reasoning of others) can more readily be facilitated in small groups (e.g., Wasik, 2008). Small groups also allow greater opportunities for students to explain their mathematical thinking and hear the reasoning and explanations of their peers (Wasik, 2008). Moreover, small groups are important learning contexts for young children. They create greater opportunities to interact one-on-one with adults and develop positive teacher-child relationships (Bowman et al., 2001), facilitate language acquisition and comprehension (Phillips & Twardosz, 2003; Wasik, 2008), and provide opportunities for students to practice academic, social, emotional, language, and self-regulation skills (Bodrova & Leong, 2007; Skinner, 2018; Vygotsky, 1978). Small group instruction provides more opportunity for individualized instruction, hands-on activities, and peer interaction than activities conducted in a whole group setting (Webb, 1991; Yackel et al., 1991). Studies have consistently shown that small group instruction has positive impacts on student achievement (Klein et al., 2008; Herrera et al., 2021; Jacob, Erickson & Mattera, 2020; Slavin et al., 2010). The extent to which textbooks differentially emphasize or suggest small group instruction likely impacts students' opportunities to learn.

Representation

Mathematical representation is a “sign, or combination of signs, characters, diagrams, objects, pictures, or graphs” which can be used to help facilitate the understanding of mathematical concepts (Mainali, 2021, p. 3). Effective mathematics instruction includes frequent use of representation to help children understand and communicate about mathematical ideas (National Council of Teachers of Mathematics, 2000; Protheroe, 2007). Constructivist theory postulates that young children gain knowledge by engaging physically with their environments (Piaget, 1970) and when children engage with manipulatives their mathematical understanding is enhanced (Greabell, 1978; Raphael & Wahlstrom, 1989; Sowell, 1989; Suydam, 1986; McNeil & Fyfe, 2012; Sarama & Clements, 2016). Hands-on experiences can play an important role in supporting learning of abstract mathematics concepts, particularly when teachers help students directly connect concrete representations to more abstract mathematical ideas (e.g., Gagatsis, 2003; Martin et al., 2007; Uttal, et al., 2009; Clements & Sarama, 2021; Roche, et al., 2021). Intentional and appropriate employment of representational strategies can have a positive impact on student learning (Canny, 1984; Carbonneau et al., 2013; Clements & Battista, 1990; Clements, 1999; Cramer et al., 2002). A 2013 meta-analysis (Carbonneau et al., 2013) which focused on the effectiveness of teaching mathematics with manipulatives found that the use of manipulatives had a positive impact on achievement when compared to teaching methods that used abstract approaches alone.

Recent reforms in mathematics education reflect these findings, stressing the importance of using manipulatives, drawings, charts, graphs, or symbols to express mathematical ideas. The NCTM (2000) in particular recommends the use of manipulatives, emphasizing the importance of representation in mathematics instruction to facilitate understanding of mathematical concepts and ideas. The CCSSM also underscore the importance of modes of representation to facilitate understanding. While standards and best practice guides emphasize the use of various representational strategies, we are not aware of any studies that focus on the extent to which textbooks vary in the degree to which they specify the use of concrete representation (e.g., the use of manipulatives or kinesthetic practices) and/or pictorial representation (e.g., the use of pictures or diagrams) in their lessons. The extent to which textbooks emphasize representation has the potential to affect students’ opportunities to learn.

Methods

Textbook Selection

We compared the following three CCSSM-aligned kindergarten math textbooks: Great Minds' *Eureka Math (2016 edition)*, McGraw-Hill's *Everyday Mathematics (2015 edition)*, and Houghton Mifflin Harcourt's *Go Math! (2015 edition)*. We selected these three textbooks because they all indicated that they were CCSSM aligned and were used widely; all three were in the top ten in terms of overall market share for elementary school math curricula (Blazar et al., 2019). There were, however, notable differences between the three. *Eureka Math* was developed to align with CCSSM. It was originally implemented throughout the state of New York and is increasingly being used across the country. *Everyday Math* represents a well-established curriculum, developed well before CCSS, which was subsequently revised to align with CCSSM. It has been in wide use across the country for several decades. Houghton Mifflin Harcourt developed *Go Math!* specifically for CCSSM, and was adopted as the core CCSSM aligned curriculum by the Council of the Great City Schools. As such, many large urban districts across the county use *Go Math!* We provide more details on each of these three textbooks below.

Eureka Math

Eureka Math was developed as part of a New York State Department of Education initiative, called EngageNY, which utilized a federally funded Race to the Top grant. It was originally developed as an open-source resource by Great Minds (formerly Common Core, Inc.), Scott Baldridge, a mathematics professor at Louisiana State University; and a team of education professionals. The curriculum was first published as *Eureka Math* in 2013. It was intentionally developed as a CCSSM-aligned curriculum.

The *Eureka Math* textbook is divided into six modules, each with individual lessons. Modules vary in terms of number of lessons, ranging from 10 to 41, with a total of 152 lessons. Each lesson follows the same daily structure, which include the following sub-sections: *Fluency Practice*, *Application Problem*, *Concept Development*, and *Student Debrief*. *Fluency Practice* activities have one of three goals: (1) Maintenance: practicing previously learned skills, (2) Preparation: targeted practice for the current lesson, or (3) Anticipation: working toward skills for upcoming lessons. *Application Problems* are short, active, individual or partnered activities that prepare students for the day's lesson. The activity may involve drawings, using concrete objects, or discussion. *Application Problems* often conclude with children comparing their work. *Concept Development* consists of a whole-group guided activity and discussion on the day's lesson. Children are often paired to practice an activity that supports the lesson. *Concept Development* ends with a short worksheet that is done individually and is aimed at reinforcing the concepts of the day's lesson. *Student Debrief* invites reflection and active processing of the lesson. Teachers are prompted to have students try out their ideas with a partner before being asked to share with the whole class. Questions are provided to help teachers guide discussion. At the conclusion of the *Debrief*, students complete an Exit Ticket that teachers can use to assess students' understanding of the concepts from the day's lesson.

Everyday Mathematics

Everyday Mathematics was developed by the University of Chicago School Mathematics Project. The kindergarten curriculum was first introduced in classrooms in 1988, with the addition of curricula for grades 1-6 over the next decade. *Everyday Mathematics* was in its third edition (2007) when the Common Core State Standards in Mathematics were released in 2010. Work began immediately to align *Everyday Mathematics* with the common core standards, and a revised third edition was released in 2011. *Everyday Mathematics'* fourth edition (2015) was used in this study. The textbook is presented in two volumes and includes a total of 117 lessons grouped into nine sections (13 lessons each). Most lessons are divided into seven sub-sections including five *Daily Routines* and two core activities (*Focus* and *Practice*). An optional unit, *Connection*, is not included in our analyses. *Daily Routines* include: (1) tracking the number of days of school on a number line and with concrete objects, with conceptual emphasis on place value, written numbers, and counting, (2) charting attendance through counting, writing numbers, collecting data, and solving problems, (3) reviewing the classroom's daily schedule and the calendar for understanding days, weeks, months and years, (4) weather and temperature for collecting, organizing and analyzing data over time, and (5) surveying classmates and collecting, recording, displaying, and discussing data. In *Focus* activities, children explore, engage in, and discuss new content. In *Practice* activities, children revisit an earlier *Focus* activity, often through a game.

Go Math!

The *Go Math!* kindergarten textbook was developed by Houghton Mifflin Harcourt and first published in 2012 to meet the Common Core State Standards. It was subsequently adopted as the CCSSM curriculum by the Council of the Great City Schools. The original version has been updated and some state specific versions have been developed. Go Math! (2015) has been used for this study. It includes 100 lessons in 12 chapters. Most lessons are divided into seven sub-sections: *Daily Routines A and B, Engage, Explore, Explain, Elaborate,* and *Evaluate*. For *Daily Routines*, teachers pose a brief question to the whole group that provides targeted practice for the upcoming lesson, practice on a previously introduced skill, and introduction or reinforcement of math vocabulary. In *Engage* an animated video introduces the learning objective of the lesson and places it in a story context after which the teacher is guided to lead a brief discussion of the main points of the video as it relates to the learning objective. In *Explore*, children solve a problem using concrete models and representational drawings. In *Explain*, the teacher leads whole-group instruction on the lesson's learning objective, with children following along in workbooks. *Elaborate* involves continued exploration of the lesson's objectives. The teacher presents problems that increase in complexity and guides problem-solving in student workbooks. In *Evaluate*, teachers are guided to have children reflect on the learning objective, demonstrating their knowledge through discussion with a partner or by using a math board.

Coding Protocol

The curriculum coding protocol was based on an adapted version of the *Classroom Observation of Early Mathematics—Environment and Teaching* protocol (COEMET; Sarama & Clements, 2008). The COEMET is an observational instrument that measures the quantity and quality of mathematics taught in preschool and elementary classrooms. It is organized around specific math activities (SMAs), defined as math activities that last for at least one minute, are intended to develop mathematical knowledge, and have a discernible topic, goal, and task. For each SMA, a primary instructional focus is identified. While

SMAs often cover multiple mathematical topics or content areas, the main mathematical focus of the activity is assigned as the primary instructional focus of the SMA. The COEMET records the length of each SMA, materials used, and instructional grouping. As described below, we adapted the COEMET into a protocol for coding curricular materials. For this study we focused on (i) the primary instructional focus, (ii) the instructional grouping, and (iii) the mode of representation used (i.e. concrete, pictorial, or abstract).

Identifying SMAs in Textbooks

As described in detail above, each of the individual textbooks we reviewed had specific sub-sections that had a specific instructional purpose and associated task. We treated each of those sub-sections as separate SMAs. For example, in *Everyday Mathematics*, the *Focus* and *Practice* sub-sections, which each have a different purpose and associated tasks, were coded as two separate SMAs. We only coded the main sub-sections of the textbooks; we did not code suggested homework activities; supplemental activities, or activities designed for particular subgroups of students (e.g., activities for English language learners).

Coding Mathematics Content

The coding protocol included the following instructional foci: *Numeral Recognition and Writing*, *Counting (Basic and Advanced)*, *Comparing and Ordering*, *Basic Subitizing*, *Composing Numbers*, *Place Value, Adding and Subtracting*, *Multiplying and Dividing*, *Shapes*, *Using Shapes to Compose Other Designs, Graphing, Motion and Spatial Sense, Measuring, Patterning, Classifying, and Fractions.* They are each described in Table 1. During training each instructional topic was described using real classroom scenarios to illustrate what it might look like in the classroom or textbook.

For each SMA, a primary instructional focus is identified. The primary instructional focus is either the topic on which a majority of the SMA is spent on or in cases where time is fairly evenly split between topics, the highest order activity. While SMAs often cover multiple math concepts, only one concept is assigned as the primary instructional focus. For example, one of *Everyday Math's* Daily Routines tracks the number of days of school on a number line and with concrete objects that are sorted into ones, tens, and hundreds buckets. Although the SMA reflects content in *Numeral Recognition and Writing*, *Basic Counting*, *Advanced Counting*, and *Place Value*, only *Basic Counting* was identified as the main instructional focus for this SMA because a majority of the SMA focused on basic counting.

Coding Instructional Grouping

Instructional grouping specifies the primary instructional grouping described for each SMA. Three types of grouping were identified (i) *Whole Group Instruction*, (ii) *Small Group Instruction*, and (iii) *Seat Work.* We considered an SMA to be *Whole Group Instruction* when the textbook instructions indicated a predominantly teacher-centered activity with the teacher giving directions, explaining a concept, or asking questions of the entire class. Grouping was coded as *Small Group Instruction* when the textbook indicated that students should work in small groups with other students. The teacher could lead the small group, facilitating the discussion, or might move from group to group allowing the children to facilitate. We classified activities where children were expected to complete tasks individually (e.g., children writing in their math journals or completing a worksheet) as *Seat Work*. While an SMA could involve more than one type of instructional grouping, the code was designated based on the grouping that was used for the majority of the activity. The following are examples of each type of instructional groupings:

- *Whole Group Instruction*: The teacher models how to make a pattern with shapes in front of the class and asks the class to make the same pattern. The teacher then calls on a student to explain and describe the pattern.
- *Small Group Instruction*: A teacher gives pairs of children numeral cards and connecting cubes. She asks students to collaborate with a partner and to use the connecting cubes as a guide to put the numeral cards in order from the smallest to the largest number. The teacher encourages the children to use strategies, such as counting on or modeling with fingers, but does not guide the activity at all times.
- *Seat Work:* All children are asked to work in their math journal independently at their own pace. The teacher circulates the room.

Coding Representation

For each SMA, the mode of representation specifies the instructional strategy used to support conceptual understanding. Three types of representation were coded (i) *Concrete Experiences*, (ii) *Pictorial Representation*, and (iii) *Abstract Representation*. *Concrete Experiences* are instructional practices that use an object that can be grasped and physically manipulated, or a kinesthetic practice (for example, two children forming their bodies into a shape), to enhance mathematical understanding. Children modeling the composition of numbers using red and yellow two-sided chips to show all the combinations of a target number would be considered a *concrete experience*. An SMA was coded as a concrete experience if most children would be afforded the opportunity to use an object or manipulative during an SMA, or if the SMA included a kinesthetic practice, such as jumping forward one hop as you count forwards by ones.

Pictorial Representations use pictures or drawings, such as circles, tally marks, or illustrations to represent concrete objects. For example, students are shown a picture of four fish, are asked to cross out two fish, and then count how many fish remain. An SMA was also coded as including pictorial representation if the teacher used an object to illustrate a mathematical concept, such as holding up a shape, but students did not handle that object. In this way, an SMA coded with a concrete experience indicates students are actually handling manipulatives during the lesson. *Abstract Representation* is when math is taught without representation or modeled with symbols only (numbers, notation, and operation symbols like + or -). For example, when children are asked to solve the problem 8 – 3 = ____. We considered an SMA to be taught at the *abstract* level *only* if a mathematical concept or problem was presented exclusively with numbers and math symbols, or if the problem asked children to engage in mental math, without the use of any concrete experiences or pictorial representations. SMAs were coded as *concrete*, *pictorial*, or *abstract* either in isolation or in any combination. An SMA in which students solved various subtraction problems using counting chips for some problems and drawings for others would be coded as using both *concrete* and *pictorial* representation. In cases where an SMA did not include any explicit guidance regarding the type of representation, *no* representation was assigned. For example, when children were asked to reflect on how they might use counting strategies to compare sets of objects without further instruction to use any type of representation, we considered the SMA to include no representation.

Coder Training and Reliability

Trained research staff, including professional research associates and graduate students, coded the curricula. All coders participated in at least a half day of training on the coding protocol. At the end of training, research staff coded a sample lesson and lead researchers compared those codes to a set of master codes. Coders were required to demonstrate 80% agreement on binary codes and 80% agreement “within 1” on Likert-scale codes with a master coder. If observers failed to meet the threshold during initial training, they received extensive feedback and coded additional lessons until they were able to use the protocol reliably. All three curricula were double coded; the group discussed any discrepancies, and a master code was determined.

Analysis

Assigning Time to SMAs

Because SMAs vary in both length and number, comparing topics or other aspects of instruction by the number of SMAs in which they were observed could over or underestimate the amount of emphasis placed on various aspects of instruction across the three textbooks. For example, we identified seven SMAs per lesson in *Everyday Math*. However, five of these SMAs encompass daily routines intended to last no longer than three minutes each. The remaining two SMAs in this textbook focus on two core activities, one lasting 30 minutes and the other 15 minutes. A comparison of content coverage at the SMA level would equally emphasize a math practice that was part of an SMA lasting 30 minutes, 15 minutes, or 3 minutes. To avoid giving undue weight to very short periods of instruction, we use the recommended length of each SMA to calculate the amount of time spent in various activities. For example, instead of counting the number of SMAs where the primary math practice was graphing, we multiply the number of SMAs where graphing was the primary math practice by the recommended length of each of those SMAs (see Appendix for the recommended lengths assigned to each SMA type across the three textbooks).

Two of the three curricula we coded (*Eureka* and *Everyday Mathematics*) provided recommended amounts of time in minutes for each SMA. Eureka recommends the following: *Fluency Practice* (~12 minutes), *Application Problems* (~5 minutes), *Concept Development* (~25 minutes), and *Student Debrief* (~8 minutes). This is equivalent to 50 minutes per lesson or approximately 126 hours of math instruction per year. *Everyday Math* suggests 10-15 minutes of instructional time for the *Daily Routines*, 20-30 minutes on *Focus*, and 10-15 minutes on *Practice*. For comparison purposes we applied the highest number of the range of minutes given for each SMA yielding about 60 minutes per lessons or approximately 123 hours of math instruction per year.

Go Math!, however, recommends half, full, or two days per individual lesson. Although there are no explicit time guidelines for Go Math! we used this information and information from another research study that conducted classroom observations of the *GoMath!* curriculum in New York City (Jacob, Engel & Erickson, et al., 2020) to assign times to each of the SMAs in *Go Math!*. Observations indicated that math instruction typically lasts approximately an hour a day for a daily lesson (consistent with the guidelines for *Eureka*, and *Everyday Mathematics*) and indicated the following typical distribution across sections: *Daily Routines* (~15 min), *Engage* (~10 min), *Explore* (~10 min), *Explain* (~10 min), *Elaborate* (~10 min), and *Evaluate* (~5 min).

Summary Statistics

To assess the mathematical content covered, we calculated the percentage of overall suggested or observed instructional time each textbook devoted to a primary instructional focus. While SMAs often cover multiple math practices, the main mathematical focus of the activity is assigned as the primary math focus, and the estimated time for each SMA was assigned to that mathematical focus. Percentages are based on total minutes of suggested instructional time in individual textbooks—*Eureka Math* (~126 hours), *Everyday Mathematics* (~123 hours), and *Go Math!* (~129 hours).

Similarly, the suggested instructional grouping and type of representation are assigned to each SMA and we calculate the percentage of overall suggested instructional time for each grouping code and representation code, based on the total minutes of instruction listed above. While an SMA can only be assigned one instructional grouping code, more than one representation code could be assigned to each SMA. Our results present aggregate numbers comparing the three textbooks. We do not conduct statistical tests of the differences between textbooks because we did not sample textbooks and we are not trying to generalize beyond the three textbooks that are included in the study. As a result, statistical inference is not necessary or appropriate (e.g. Healy, 1999).

Results

Mathematical Content Coverage

Table 2 shows the percentage of time each textbook suggested teachers spend on various mathematical topics, based on the *primary instructional focus* assigned to an SMA. To highlight some of the observed differences across the three textbooks, we show the percentage of time devoted to topics related to *Numbers and Operations* (e.g., counting, composing numbers, addition and subtraction) and the time devoted to other math practices such as geometry, measurement, patterning, etc., separately. This distinction is important because the CCSSM recommend that at least 65%, and preferably closer to 85%, of instructional time for kindergarten be focused on *Numbers and Operations* (Achieve the Common Core, n.d.). We organize the collection of topics related to *Numbers and Operations* into two categories: (1) *Foundational* and (2) *Advanced*. *Foundational* practices include *Basic Counting, Basic Subitizing*, and *Numeral Recognition and Writing*—topics for which children entering kindergarten may have prior knowledge. *Advanced* practices include *Advanced Counting, Comparing and Ordering, Composing Numbers, Place Value, and Adding and Subtracting*. These practices represent topics that are likely to be new to many incoming kindergarten students. Other math practices include *Shapes*, *Using Shapes to Compose Other Designs*, *Graphing*, *Motion and Spatial Sense*, *Measurement*, *Patterning*, *Fractions*, and *Classifying*.

Table 2 includes several notable findings. First, in all three textbooks over 60% of suggested instructional content focused on *Numbers and Operations.* This is consistent with CCSSM kindergarten recommendations, which indicate that at least 65%, and preferably closer to 85%, of instructional time for kindergarten be focused on these topics (Achieve the Common Core, n.d.). We find that *Go Math!* devotes the most time to *Numbers and Operations* (84.4%), with *Eureka Math* a close second (77.7%). *Everyday Mathematics* places the least emphasis on *Numbers and Operations* (61.4%) among the three. With respect to other math practices, *Eureka Math* places a strong emphasis on *Measurement* (over 10% of suggested instructional time), while *Everyday Mathematics* emphasizes *Shapes, Measurement* and *Graphing*, with a particular emphasis on *Graphing* which constitutes almost 14% of the instructional content focus. In contrast, *Graphing* was never identified as the primary math practice in *Eureka Math*, and represented only 1% of suggested instructional time in *Go Math!*. *Classifying, Patterning, Motion and Spatial Sense* receive little attention across all three textbooks (less than 5% of suggested instructional time).

Within *Numbers and Operations*, we find a substantial difference in the suggested instructional time devoted to foundational topics relative to advanced topics. *Go Math!* suggests devoting ~30% of instructional time to foundational topics like *Basic Counting*, *Basic Subitizing* and *Numeral Recognition*, while *Eureka Math* and *Everyday Mathematics* suggest devoting around 20% of instructional time to these topics. We also see differences in advanced topics that are likely new to children. *Go Math!*, suggests spending almost twice as much instructional time (23%) on practices related to *Adding and Subtracting* compared to the other two textbooks, while *Eureka Math* emphasizes *Composing Numbers*, with 31% of suggested instructional time devoted to this practice. In contrast, *Composing Numbers* receives primary emphasis only 8% and 11% of the time in *Everyday Mathematics* and *Go Math!*, respectively.

We also explored topic sequencing within the textbooks. Figures 1-3 outline the sequencing of primary math practices for each textbook. The figures show when a topic is introduced and how frequently the textbook returns to that topic throughout the year.

In *Eureka Math* (Figure 1) we can see that the three *Numbers and Operations* practices, *Basic Counting (*foundational*), Comparing and Ordering*, and *Composing Numbers* (advanced) are introduced early and are addressed repeatedly throughout the school year. Further math practices like *Classifying, Motion and Spatial Sense,* and other foundational *Numbers and Operations* practices like *Numeral Recognition,* and *Recognizing Quantity without Counting (Basic Subitizing)* are introduced in the first half of the year. However, they are a primary focus for only a short period of time and are not revisited regularly during the latter half of the year. *Measuring* is also introduced in the first half of the year but receives coverage over a longer period of time, and continues into the third quarter of the year. Figure 1 also shows that the advanced practice *Adding and Subtracting* receives brief coverage early and then more frequently in the second half of the school year. *Place Value* is introduced in the last quarter of the year.

Figure 2 shows the sequencing of the primary mathematical foci for *Everyday Mathematics*. In contrast to the other two textbooks, all topics (with the exception of *Place Value* and *Patterning*) are introduced in the first half of the school year and returned to repeatedly throughout the year. Three math topics, representing a mix of foundational, advanced, and other math practices, receive daily attention: *Basic Counting*, *Comparing and Ordering*, and *Graphing.* It is important to note that although *Place Value* was rarely coded as a primary focus in later lessons, in the Daily Routines, it was frequently addressed, just in a less salient way, and was therefore not coded as the primary instructional focus.

In *Go Math!* five topics, representing a mix of foundational and advanced practices under *Numbers and Operations*, are introduced early and revisited often throughout the year—they are *Basic Counting*, *Comparing and Ordering*, *Composing Numbers*, *Adding and Subtracting,* and *Numeral Recognition and Writing* (Figure 3*). Recognizing Quantity without Counting (Basic Subitizing)* is also introduced early but receives only brief coverage. Many other practices not classified under *Numbers and Operations,* like *Classifying, Measuring, Motion and Spatial Sense, Place Value, Shapes,* and *Graphing* are introduced in the last quarter of the year and receive less time.

Instructional Grouping

Table 3 shows variation across the three textbooks in terms of suggested instructional grouping. All three textbooks employ whole group instruction as the primary instructional modality—over 70% of activities in each textbook are facilitated in a whole group setting. However, the textbooks differ in their use of small group activities. While both *Everyday Mathematics* and *Eureka Math* recommend that 15-20% of their activities occur in small groups, only about 7% of *Go Math!* activities are suggested as small group activities. Just over 20% of all *Go Math!* activities involve seat work (children working individually and independently on an activity at their seat) compared with fewer than 10% of *Eureka Math* and *Everyday Mathematics* activities.

Representation

Table 4 indicates the percentage of activities that involve *concrete*, *pictorial,* and/or *abstract* representation. Activities often involve multiple forms of representation so the categories are not mutually exclusive. Almost all activities across all three textbooks employed some form of representation. Both *Eureka Math* and *Everyday Mathematics* had fewer than 1% of activities that used no representation. *Go Math!* had the largest number of activities with no representation (around 8%). Most of the activities with no representation in *Go Math!* were reflective in nature and did not include explicit reference to any type of representation (e.g., asking students to name objects that are shaped like a triangle). All three textbooks employed some form of pictorial representation in a majority of activities, 84.3% of the time in *Eureka Math*, 77.7% in *Go Math!*, and 75.1% in *Everyday Mathematics*. Concrete representation was used less frequently (between 43% and 55% of the time across the three textbooks). The use of *abstract* representation--presentation of mathematical problems exclusively with numbers and math symbols--was consistently low across textbooks: 8.3% for *Go Math!*, 6.7% for *Everyday Mathematics*, and 4.7% for *Eureka Math*.

Table 5 presents results from an analysis of representation separately by primary math practice, focusing on the four math practices that were emphasized most in the three textbooks: *Counting, Comparing and Ordering, Composing Numbers,* and *Adding and Subtracting*. While overall the emphasis on concrete experiences ranged from 43% to 55% as displayed in Table 4, some textbooks embedded concrete experiences more often in certain math practices than others. For example, *Everyday Mathematics* used concrete experiences in lessons on *Counting* and *Comparing and Ordering* less than a third of the time. *Go Math!* on the other hand, specified concrete experiences for lessons with a focus on *Counting* and *Comparing and Ordering* 44.2% and 44.0% of the time, respectively. For *Eureka Math* concrete experiences were specified in *Counting* and *Comparing and Ordering* lessons 52.7% and 49.4% of the time, respectively. In contrast, *Everyday Mathematics* used concrete experiences 72.5% of the time when covering *Composing Numbers* and 58.1% of the time in *Adding and Subtracting* activities, substantially more time than either of the other textbooks.

Discussion

This study analyzed three widely used kindergarten math textbooks that were written to align with the CCSSM. Results provide insight into what content is emphasized, when that content is introduced, and how often it is revisited throughout the school year, as well as information on instructional grouping, and modes of representation. Textbooks offer one avenue by which standards, like the CCSSM, are translated into educational practice and have been shown to predict both learning opportunities and student skill development (Agodini & Harris, 2010; Bhatt & Koedel, 2012; Kane et al., 2016; Koedel et al., 2017; Son & Diletti, 2017; Van de Walle et al., 2010). However, content analysis of the type undertaken here, which might reveal the underlying mechanisms by which textbooks impact student outcomes is rare. Research about the effectiveness of kindergarten mathematics curricula is especially limited. While some have assessed the degree to which various mathematics textbooks are aligned to the Common Core standards (e.g., Polikoff, 2015), the analysis presented here highlights the ways in which textbooks that are aligned to the same set of standards can vary in substantial ways and in ways that have important implications for students' opportunities to learn.

Mathematical Content

Consistent with CCSSM recommendations, the three textbooks reviewed here all emphasized primary math topics related to *Numbers and Operations*, and all focused most on *Basic Counting* and *Comparing and Ordering*. However, content emphasis also varies in important ways. *Go Math!* places a greater emphasis, relative to the other two textbooks, on more foundational math practices that research has shown many children may have already mastered upon entry into kindergarten (Engel et al., 2013), including *Numeral Recognition*, *Basic Counting* and *Subitizing*. The difference is particularly striking when comparing *Go Math!* and *Eureka Math*. *Eureka Math* devotes half as much time to these topics as *Go Math!*. Additional exposure to practices around basic counting for children who already possess these skills negatively predicts math achievement (Engel et al., 2013; Engel et al., 2016). On the other hand, *Go Math!* also devotes more time to addition and subtraction than the other two coded textbooks.

We also find that *Everyday Mathematics* devotes substantially more time to math practices beyond *Numbers and Operations* (e.g., geometry, measurement, graphing, patterning, etc.) than the other two textbooks reviewed. Almost 40% of instructional time in *Everyday Mathematics* is devoted to these math practices, with the greatest emphasis placed on *Shapes* and *Graphing.* Devoting time to other mathematical concepts builds overall mathematical proficiency and allows children with different learning styles or orientations to demonstrate their competency (Clements & Sarama, 2021). Claessens and Engel (2013) find, for example, that both early pattern recognition and measurement skills are predictive of outcomes in eighth grade. Yet, one goal of the CCSSM was to trade breadth for depth, and to shift away from covering a large number of topics without deep understanding. The CCSSM identify *Number and Operations* as the key content focus for kindergarten. By continuing to emphasize a relatively broader range of topics, *Everyday Mathematics* may not be fully embracing one of the key tenants of the standards.

Topic sequencing also differed substantially across the textbooks; the sequencing in both *Go Math!* and *Eureka Math* reflect a mastery approach, in which students spend extended time on one topic, with the hope that they will master a given level of thinking before moving to the next topic. *Everyday Mathematics* reflects a spiraling approach, in which topics are returned to repeatedly throughout the year, ideally with greater sophistication each time. The mastery approach reflects the CCSSM emphasis on depth over breadth. However, this approach also means that some math practices are not introduced until the end of the year. In *Go Math!*, for example, not only do topics related to *Shapes, Motion and Spatial*

Sense, Measuring, Classifying, and *Graphing*—those not classified as *Numbers and Operations*—receive limited emphasis, they are also only introduced in the last units of the textbook. Teachers, who face competing demands on their classroom time, often do not complete instruction on all units of a textbook during a school year (Flanders, 1994). As a result, topics that are introduced at the end of the year may not be covered at all. On the other hand, the spiraling approach may not allow some students sufficient time to engage with one topic before moving on to the next (Snider, 2004).

Instructional Grouping

All three textbooks emphasized whole group, teacher-centered instruction over small group activities. This was especially so in *Go Math!* where around 7% of the instructional time indicated the use of small groups. Effective mathematics instruction allows ample opportunity for students to share mathematical ideas and engage with their peers (National Council of Teachers of Mathematics, 2000; Protheroe, 2007; Sarama & Clements, 2009; Skinner, 2018; Stright & Supplee, 2002; van Oers, 2010). Differentiation, peer interaction, and math talk are all facilitated when students work in small groups (Jacob, Erickson, & Mattera, 2020; Wasik, 2008). Textbooks that encourage small group instruction more frequently may therefore increase students' opportunities to learn.

Representation

We found limited variation across textbooks regarding the use of different forms of representations. All three textbooks afforded extensive opportunities for instruction using *pictorial* representation (over ¾ of instructional time), with *concrete* experiences included around half of the time. However, not all textbooks used representation in the same way. The frequency with which the three textbooks emphasized concrete learning opportunities differed with respect to *Basic Counting*, *Comparing and Ordering, Addition and Subtraction* and *Composition of Numbers*, with *Everyday Mathematics* emphasizing concrete experiences substantially less in lessons on *Basic Counting* and *Comparing and Ordering*, and substantially more than the other two textbooks in lessons on *Addition and Subtraction* and *Composition of Numbers*. This finding is surprising, since concrete representation is important both for supporting the development of counting skills and for facilitating understanding related to addition and subtraction (Carpenter et al., 2017). The type of representation curricula use and how they move students from using one type to another warrants further investigation.

Implications for Practice

These findings have several implications for district leaders and others making important curricular decisions. First, leaders should not assume that all CCSSM aligned curricula are the same. Findings here suggest that they can differ in important ways that have implications for students' opportunities to learn. Selecting the most appropriate textbook for a particular district or school requires understanding what incoming kindergarteners are likely to know and be able to do. Is an emphasis on more foundational skills warranted, or will most students arrive already knowing these skills and benefit from more time on more advanced mathematical topics? The findings also suggest that pacing is important—the CCSSM for kindergarten include both geometry and measurement. Since some curricula leave these topics to the end of the year, it is important that teachers have sufficient time to get through the entire textbook, or that some units are moved to ensure that they are covered, if students are going to be exposed to the full range of skills recommended by the standards.

Research is also clear that small group instruction is a powerful instructional tool. If a textbook places limited emphasis on small group instruction, teachers and curriculum leaders should seek out other ways to incorporate small group work into their math instruction. In some cases, curricula have optional suggestions for small group activities that could be incorporated; in other cases, supplemental activities or curricula might be needed. However, without more specific guidance, teachers will be inclined to focus their efforts on the key components and topics emphasized by the textbook, leaving student and teacher with few opportunities to interact in a smaller setting around mathematical topics. A similar argument can be made with respect to representation; teachers are likely to only use the type of representation presented in the curriculum, limiting student exposure to other modes of representation.

Limitations

While this study reveals important differences across these three widely used kindergarten mathematics textbooks, it also has some limitations. First, our analyses is purely descriptive in nature—

we describe similarities and differences across these three textbooks, but do not attempt to evaluate their relative effectiveness.

Second, our content analysis is limited to the primary instructional focus of each SMA and does not include other, concurrent, math topics covered within a single SMA. All three textbooks provide opportunities to learn various mathematical topics within SMAs that were not identified as the primary focus. For example, an SMA whose primary instructional focus was addition and subtraction might also emphasize composition of numbers as one strategy for deriving answers. Similarly, our analysis only focuses on the instructional grouping that was identified as constituting the majority of an SMA, and may have missed other instructional groupings that did not constitute the majority.

Our content analysis is also limited to what and how often math topics are covered. We do not make qualitative distinctions regarding how particular topics are introduced, whether the introduction of math practices follows a clear learning trajectory, how connections between topics are encouraged, or how children are engaged to develop their understanding of mathematical problems. Similarly, our analysis of the different types of representation used in these three textbooks focused on the presence, not the effectiveness, of concrete, pictorial, or abstract representation within an SMA. Concrete representation in and of itself does not facilitate learning (Ball, 1992; Baroody, 1989; Clements & Sarama, 2021). Effective use of manipulatives requires skill and intent and the manipulative must meaningfully connect to students' existing knowledge in a way that facilitates understanding of the underlying concept it is supposed to represent (Baroody, 1989). In this analysis we did not code how representation was introduced or connected to concept development and furthering understanding. Future research in this area is warranted.

There are also some general limitations with regard to comparative research as it relates to textbook analysis. Textbooks vary in the length and number of pages of instructions, math problems, lessons, and more, making comparison across textbooks challenging. We used the suggested length of each SMA to compare content coverage and other variables across textbooks. However, doing so required making some assumptions regarding the length of specific activities, especially with respect to Go Math! which does not provide a recommended length of time for the SMAs.

Finally, we coded a single edition of each textbook and some have published newer editions that are not reflected in our findings. However, our interest is not on focusing on the three curricula analyzed here. Our interest lies more broadly in exploring whether and how different curricula that are intending to align with a common set of standards vary within a single grade.

Conclusion

The purpose of this study was to explore variation in suggested instructional approaches across three CCSSM-aligned kindergarten textbooks and to describe similarities and differences in terms of content coverage and sequencing, use of representational strategies, and instructional grouping. All three of these factors have the potential to influence students' opportunities to learn. Our analysis finds that although all three textbooks are CCSSM-aligned and have many similarities, they also differ in potential important ways.

All three textbooks: (i) place the most emphasis on math practices in *Numbers and Operations*, reflecting the CCSSM's focus in this area and a promotion of depth over breadth; (ii) are similar in terms of specifying relatively frequent use of *concrete* and/or *pictorial* representation for solving math problems; and (iii) most often guide teachers toward whole class, teacher-centered instruction. Beyond these commonalities, we identify notable differences. *Eureka Math* places a stronger emphasis on *Composing Numbers* and *concrete* experiences. *Everyday Mathematics* covers more mathematical topics, and employs a spiraling strategy in which instruction on various mathematical practices is repeated consistently over time, rather than focusing on one practice for an extended period of time. *Go Math!* emphasizes depth over breadth, covering fewer topics at greater length. *Go Math!* also places greater emphasis on more foundational math practices, and favors whole class instruction and seat work over small group instruction.

Future research should build on the results presented in the current study in several ways. First, an implementation study exploring the extent to which these curricula as enacted (e.g., classroom observations) aligns with the intended curriculum (the textbook) in terms of content coverage, topic sequencing, instructional grouping, and opportunities for representation would help to inform how reliable these textbooks are as proxies for students' opportunities to learn. Second, exploring the relationship between textbooks, implementation fidelity, and students' short- and long-term mathematical skills development would further our understanding of the degree to which textbooks are a malleable aspect of classroom instruction that can influence student outcomes. Another possible area for future research is an exploration of whether and how unique contextual factors within school districts drive the choice of curriculum. For example, how should knowledge of students' prior exposure to mathematics, expectations of knowledge in later grades, class size, or teachers' experiences and needs inform which textbook may be most effective to address the needs of students in a particular district or school?

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Table 1

Primary Math Practices and Descriptions

Math Practice	Description
Numeral Recognition and Writing	Reading and writing numerals. For example, practicing writing the number five.
Basic Counting	Counting forward by 1s and one-to-one correspondence. For example, counting the number of snap cubes in a pile.
Advanced Counting	Counting backwards and skip counting by any number. For example, counting by 5s or 10s.
Comparing and Ordering	Comparison and sequencing activities that answer questions such as: Which has more? Which comes first? What is one more? What is one less?
Basic Subitizing (Recognizing Quantities without Counting)	Without counting, instantly knowing how many items are in a small set of items. For example, recognizing the number of dots on one side of a die without counting.
Composing Numbers	Producing number combinations that “make” a given number. For example, 8 is composed of [7,1], [6,2], [5,3], and so on.
Place Value	Recognizing a whole number as a sum of other numbers in terms of its base 10 expansion. For example, 24 is a set of two 10s and four 1s.
Adding and Subtracting	Adding numbers to determine the sum, or subtracting numbers to determine the difference.
Multiplying and Dividing	Combining groups of equal size, or splitting a set into groups of equal size (“fair sharing”). For example, if there are three children, and I have six crackers, how many crackers does each child get?
Shapes	Identifying, describing, constructing, comparing, and matching shapes. For example, describing the difference between a rectangle and a square.
Using Shapes to Compose Other Designs	Using 2-D or 3-D shapes to make a picture of another shape, or decomposing 2-D or 3-D shapes into their parts. For example, using two triangles to create a rectangle.
Graphing	Data collection, graphing data (line plots, tally charts, bar charts, etc.), and reading and interpreting data presented in a graph.
Motion and Spatial Sense	Using position words such as up/down, next, above, beside, left/right, and under/over. Turning, flipping, or sliding shapes to see that it is identical to another shape (for example, when working a puzzle).
Measuring	Using a standard unit to measure length, weight, volume, time, or area. Ordering objects by length, weight, or volume.
Patterning	Recognizing, copying, or extending patterns. For example, what comes next in the following sequence? Red, blue, blue, red, blue…
Classifying	Organizing items into groups based on common characteristics, such as by shape, color, and size. For example, sorting snap cubes by color.
Fractions	Identifying parts of a whole. For example, recognizing that ¼ of a circle is shaded.

Table 2

Content Coverage Across Textbooks by Primary Math Practice (% of total time)

	Eureka Math (%)	Everyday Mathematics (%)	Go Math! (%)
Numbers and Operations - Foundational:			
Numeral Recognition and Writing	1.8	5.7	6.3
Basic Counting (Counting forward by 1s, 1:1 correspondence)	14.6	12.1	22.5
Subitizing	0.4	1.8	1.0
Total	16.8	19.6	29.8
Numbers and Operations - Advanced:			
Advanced Counting (skip counting, counting backwards)	0.6	3.0	2.6
Comparing and Ordering	16.1	17.7	17.2
Composing Numbers	31.2	8.1	11.4
Place Value	1.7	0.4	0.4
Adding and Subtracting	11.3	12.6	23.0
Total	60.9	41.8	54.6
Other Math Practices:			
Shapes	7.4	9.3	7.6
Using Shapes to Compose other Designs	0.5	2.2	0.6
Graphing	0.0	13.8	1.0
Motion and Spatial Sense	0.2	2.8	1.6
Measurement	10.5	7.1	2.9
Patterning	0.0	0.8	0.0
Fractions	0.0[a]	0.0	0.0
Classifying	3.8	2.4	2.0
Total	22.4	38.4	15.7

Note. SMAs often cover multiple math practices. The main mathematical focus of the activity is assigned as the primary math practice, and the estimated time for the SMA is associated with the primary math practice. Percentages are based on total minutes of instructional time which was estimated based on the suggested time in individual textbooks—Eureka Math (~126 hours), Everyday Mathematics (~123 hours), Go Math! (~129 hours). The percentages of total time spent on Numbers & Operations are 77.7% for Eureka Math, 61.4% for Everyday Mathematics, and 84.4% for Go Math!. Column percentages might not add up to 100% due to rounding.

[a] In one instance *Fractions* was identified as the primary math practice in Eureka Math (0.04% of time).

Table 3

Instructional Grouping (% of total time)

	Eureka Math (%)	Everyday Mathematics (%)	Go Math! (%)
Seat Work	9.4	7.3	20.7
Small Group	19.4	15.1	7.1
Whole Group	71.2	77.7	72.2

Note. Percentages are based on total minutes of instructional time which was estimated based on the suggested time in individual textbooks —Eureka Math (~126 hours), Everyday Math (~123 hours), Go Math! (~129 hours).

Table 4

Types of Representation Across Textbooks (% of total time)

	Eureka Math (%)	Everyday Mathematics (%)	Go Math! (%)
Concrete	54.7	46.3	43.1
Pictorial	84.3	75.1	77.7
Abstract	4.7	6.7	8.3
None	0.9	0.8	7.9

Note. Percentages are based on total minutes of instructional time based which was estimated based on the suggested time in individual textbooks—Eureka Math (~126 hours), Everyday Mathematics (~123 hours), Go Math! (~129 hours). Column percentages do not add up to 100% because some lessons contain more than one representation.

Table 5

Distribution of the Use of Concrete, Pictorial, and Abstract Representation by Primary Math Practice (% of total time for specific math practice)

	Eureka Math (%)	Everyday Mathematics (%)	Go Math! (%)
Counting			
Concrete	52.7	32.1	44.2
Pictorial	77.9	78.6	76.0
Abstract	2.3	4.0	7.6
Comparing and Ordering			
Concrete	49.4	27.5	44.0
Pictorial	82.7	79.4	70.0
Abstract	4.2	11.4	7.6
Composing Numbers			
Concrete	53.7	72.5	66.8
Pictorial	86.5	80.0	87.0
Abstract	0.6	5.0	4.8
Adding and Subtracting			
Concrete	36.6	58.1	37.1
Pictorial	82.0	62.9	77.2
Abstract	13.6	29.0	9.7

Note. Table 5 compares the distribution of representation as identified by the primary math practice assigned to an SMA. While SMAs often cover multiple math practices, the main mathematical focus of the activity is assigned as the primary math practice, and the time for the SMA is associated with the primary math practice. Percentages are based on total minutes of instructional time by primary math practice that was estimated using the suggested time in individual textbooks. Percentages do not add up to 100% because some lessons contain more than one representation.

Table

Appendix

SMAs by Textbook and Time

SMA	Time (minutes)
Eureka Math	
Fluency Practices	12
Application Problem	5
Concept Development	25
Student Debrief	8
Everyday Mathematics	
Daily Routines	15
Focus	30
Practice	15
Go Math!	
Daily Routines	15
Engage	10
Explore	10
Explain	10
Elaborate	10
Evaluate	5

Appendix. The Eureka Math textbook outlines the exact number of instructional minutes for each specific math activity. The numbers in the Estimated Time column for Eureka Math represent an approximate average across SMAs of that type. The total instructional time of Eureka Math is approximately 126 hours per year. Most often Eureka Math provides three fluency practices per lesson, and the Estimated Time in this table is based on three fluency practices. Sometimes, however, Eureka Math provides fewer than three or more than three fluency practices in a given lesson. The Everyday Mathematics textbook outlines a time span for each specific math activity. For comparison reasons we have applied the highest number of the range of minutes provided for each SMA, resulting in a total instructional time in Everyday Math of approximately 123 hours per year. The specific math activity “Connections” in Everyday Math was not considered for this analysis because it was optional. Go Math! outlines instructional times for whole lessons in days rather than minutes. For comparison reasons we have applied an educated estimate for how days can be translated into minutes of instructional time, based both on the average number of minutes Everyday Math and Eureka Math allocate to a daily math lesson, and on observations of classrooms that use the Go Math! curriculum. The resulting estimated instructional minutes for each Go Math! SMA can be found in the Time column. Thus, the total instructional time of Go Math! is approximately 129 hours per year. Most often Go Math! provides two daily routine practices per daily lesson, and the Estimated Time in this table is based on two daily routines. Sometimes, however, Go Math! provides only one or more than two daily routines in a daily lesson.

Figure 1

Topic Sequencing in Eureka Math

Note. This figure shows how the instruction of primary math practices (at the SMA-level) is distributed throughout the school year. We used the estimated length of time of each SMA (Table A1) to estimate at what time during the school year primary math practices are introduced and revisited, as represented by a dot on the figure. The primary math practices are not weighted for this overview. A single dot in the figure indicates only that the math practice was identified as a primary math practice in an SMA estimated to be conducted at that point in time; it does not reflect the length of the SMA.

Figure 1

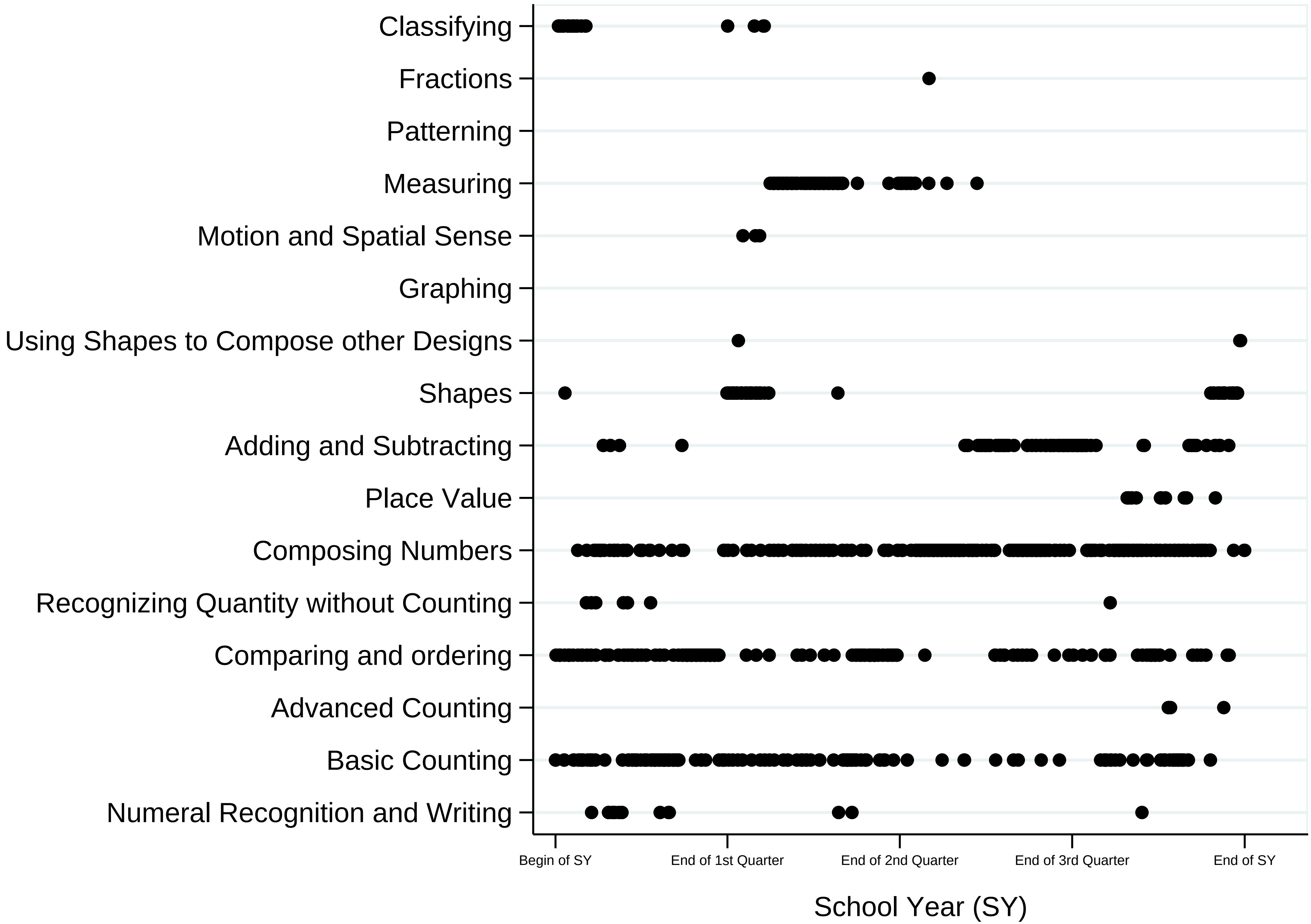
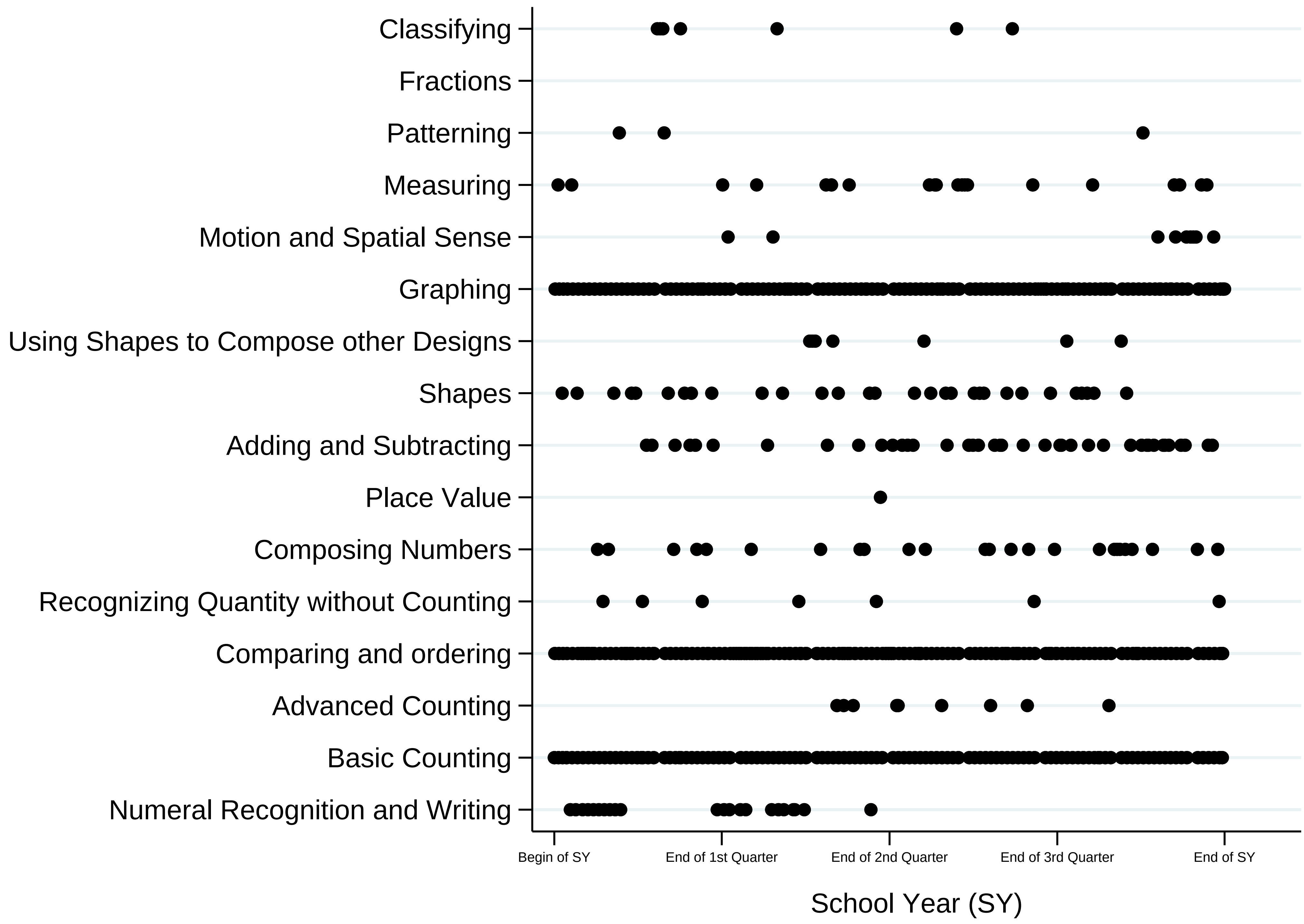

Classifying
Fractions
Patterning
Measuring
Motion and Spatial Sense
Graphing
Using Shapes to Compose other Designs
Shapes
Adding and Subtracting
Place Value
Composing Numbers
Recognizing Quantity without Counting
Comparing and ordering
Advanced Counting
Basic Counting
Numeral Recognition and Writing
Begin of SY
End of 1st Quarter
End of 2nd Quarter
End of 3rd Quarter
End of SY
School Year (SY)

Figure 2

Topic Sequencing in Everyday Mathematics

Note. This figure shows how the instruction of primary math practices (at the SMA-level) is distributed throughout the school year. We used the estimated length of time of each SMA (Table A1) to estimate at what time during the school year primary math practices are introduced and revisited, as represented by a dot on the figure. The primary math practices are not weighted for this overview. A single dot in the figure indicates only that the math practice was identified as a primary math practice in an SMA estimated to be conducted at that point in time; it does not reflect the length of the SMA.

Figure 2



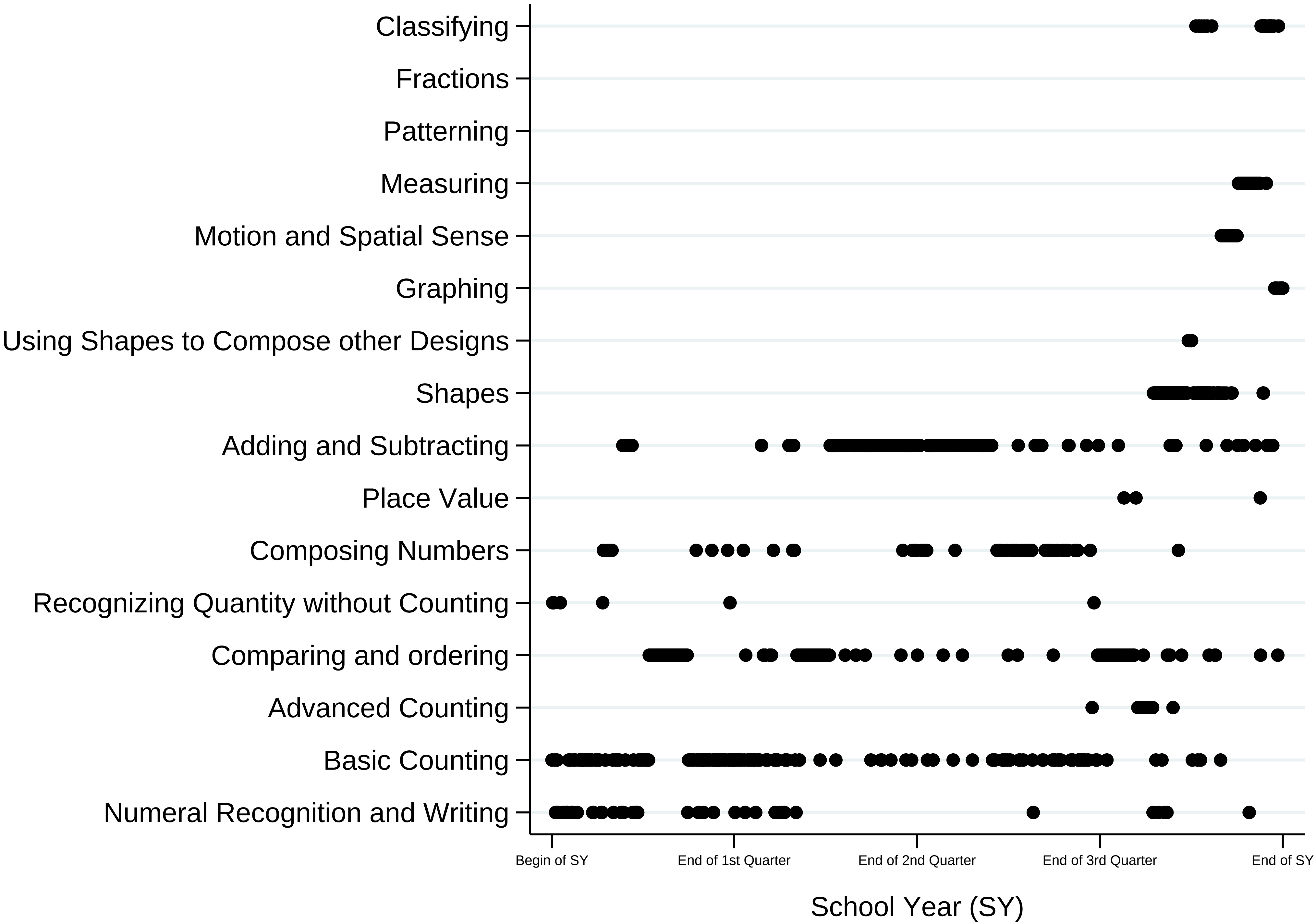
Classifying
Fractions
Patterning
Measuring
Motion and Spatial Sense
Graphing
Using Shapes to Compose other Designs
Shapes
Adding and Subtracting
Place Value
Composing Numbers
Recognizing Quantity without Counting
Comparing and ordering
Advanced Counting
Basic Counting
Numeral Recognition and Writing
Begin of SY
End of 1st Quarter
End of 2nd Quarter
End of 3rd Quarter
End of SY
School Year (SY)

Figure 3

Topic Sequencing in Go Math!

Note. This figure shows how the instruction of primary math practices (at the SMA-level) is distributed throughout the school year. We used the estimated length of time of each SMA (Table A1) to estimate at what time during the school year primary math practices are introduced and revisited, as represented by a dot on the figure. The primary math practices are not weighted for this overview. A single dot in the figure indicates only that the math practice was identified as a primary math practice in an SMA estimated to be conducted at that point in time; it does not reflect the length of the SMA.

Figure 3


Classifying
Fractions
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Basic Counting
Numeral Recognition and Writing
Begin of SY
End of 1st Quarter
End of 2nd Quarter
End of 3rd Quarter
End of SY
School Year (SY)